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Induced mass in $N = 2$ super Yang-Mills theories

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Abstract: The masses of the matter fields of $N = 2$ Super-Yang-Mills theories can be defined as parameters of deformed supersymmetry transformations. The formulation used involves central charges for the matter fields. The explicit form of the deformed supersymmetry transformations and of the invariant lagrangian in presence of the gauge supermultiplet are constructed. This works generalizes a former one, due to the same authors, which presented the free matter case.

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Contents

1. Introduction

As it is well known [1, 2], a superspace description of supersymmetric theories has to be formulated in terms of *constrained* superfields. In the case of gauge theories with $N = 2$ extended supersymmetry, the constraints are those on torsion and curvature of the gauge superfield [3]–[6] and on the matter superfields (Fayet hypermultiplets) [7]–[10], inclusively on their central charge dependence. Our aim is to show that a generalization of the Sohnius central charge constraint [9] for the hypermultiplet generates a mass for the corresponding matter particles, this mass appearing as a parameter Λ in the generalized central charge constraint and in the resulting deformed supersymmetry transformations of the component fields. A preliminary version of this work [11] dealt with the case of the free hypermultiplet only.

We shall extend here the construction to the general case of matter hypermultiplets coupled to gauge fields. We shall present the (deformed) supersymmetry transformations of the component fields, as well as the construction of the invariant action, using for the matter part an algorithm due to Hasler [12]. The gauge supermultiplet component field content corresponds to a gauge of the Wess-Zumino type, the gauge degrees of freedoms being reduced to those of a usual Yang-Mills theory.

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Our result reproduces in a purely algebraic way that of [13, 14]. The latter indeed showed that the masses of the matter particles may be generated by coupling the matter fields to a constant abelian super-Yang-Mills field strength, the values of the masses being proportional to the value of this field strength. Thus, the mass generation via a generalized central charge constraint as proposed in the present paper offers an alternative way of generating the masses, with the parameter Λ replacing the constant field strength. The interest of this alternative way is that it appears more natural, being purely algebraic.

Independent of these considerations, the explicit construction of the theory with central charge dependent matter supermultiplets as performed here, presents its own interest and, in the best of our knowledge, has not yet been shown in the literature.

The plan of the paper is the following. After recalling some basic notions for $n = 2$ superspace in section 2 and reviewing the implementation of the generalized Sohnius constraint in the free case [11] in section 3, we consider the case of the coupling with a gauge supermultiplet in section 4. Our conclusions are presented in section 5.

2. $N = 2$ Central charge superspace

Flat $N = 2$ superspace with central charge[1] [9, 10, 4, 5, 6] will be described by the coordinates $\{X^A\} = \{x^a, \theta^i_\alpha, \bar\theta_{i\dot\alpha}, z, \bar z\}$, respectively the space-time coordinates x^a, a complex Weyl spinor - isospinor θ^i_α and a complex central charge z. The spinor coordinates θ are Grassmann (i.e. anticommuting or "fermionic") numbers, the remaining ones are ordinary (i.e. commuting or "bosonic") numbers, so the manifold coordinates satisfy the (anti)commutation rules:

$$X^A X^B = (-)^{ab} X^B X^A \tag{2.1}$$

with the grading $a = 0$ if X^A is bosonic, and $a = 1$ in the fermionic case.

$N = 2$ supersymmetry is defined by the Wess-Zumino superalgebra [8, 1]

$$(\mathcal{P}_A, \mathcal{P}_B\} = T_{AB}^{C}\mathcal{P}_C \,, \tag{2.2}$$

where $\mathcal{P}_A = \{P_a,\, Q^i_\alpha,\, \bar Q_{i\dot\alpha},\, Z,\, \bar Z\}$ is the set of infinitesimal generators: the translations P_a ($a = 0, \cdots, 3$), the supersymmetry generator Q^i_α, its hermitian conjugate $\bar Q_{i\dot\alpha}$ (α and $\dot\alpha = 1, 2 =$ spin indices; $i = 1, 2 =$ SU(2) (isospin) index) and the complex central charge generator Z. Under Lorentz transformations, P_a transforms as a vector; Q and $\bar Q$ as Weyl spinors, respectively in the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations; Z and $\bar Z$ transform as scalars. Moreover Q and $\bar Q$ transform as doublets of the isospin group SU(2), the remaining generators being singlets.

The generators P, Z and $\bar Z$ are bosonic, whereas Q and $\bar Q$ are fermionic. Accordingly, the bracket $(\cdot,\cdot\}$ in the l.h.s. of (2.2) is an anticommutator if both entries are fermionic, and a commutator otherwise.

Finally, the structure constants of the superalgebra (2.2) – the "torsions" – are given by:

$$T_{\alpha\ \beta}^{i\ j\ z} = 2i\varepsilon^{ij}\varepsilon_{\alpha\beta} \,, \qquad T_{\dot\alpha\ \dot\beta}^{i\ j\ \bar z} = 2i\varepsilon^{ij}\varepsilon_{\dot\alpha\dot\beta} \,, \qquad T_{\alpha\ \dot\beta}^{i\ j\ a} = -2i\varepsilon^{ij}\sigma^a_{\alpha\dot\beta} \,, \tag{2.3}$$

all the other torsion coefficients vanishing.

[1]Our notations and conventions are given in the appendix.

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Representations of the Wess-Zumino superalgebra (2.2) are defined as superfields. A superfield $\phi(X)$ is a function in superspace transforming under the generators of the superalgebra as follows:

$$
\begin{aligned}
P_a\phi &= \partial_a\phi\,, \\
Q^i_\alpha\phi &= \left(\partial^i_\alpha - i\sigma^a_{\alpha\dot\beta}\bar\theta^{i\dot\beta}\partial_a + \theta^i_\alpha\partial_z\right)\phi\,, \qquad & \bar Q_{i\dot\alpha}\phi &= \left(-\bar\partial_{i\dot\alpha} + i\theta^\alpha_i\sigma^a_{\alpha\dot\alpha}\partial_a - \bar\theta_{i\dot\alpha}\partial_{\bar z}\right)\phi\,, \\
Z\phi &= \partial_z\phi\,, & \bar Z\phi &= \partial_{\bar z}\phi\,.
\end{aligned}
\tag{2.4}
$$

with $\partial_a = \partial/\partial x^a$, $\partial^i_\alpha = \partial/\partial\theta^\alpha_i$, $\bar\partial_{i\dot\alpha} = \partial/\partial\bar\theta^{i\dot\alpha}$, etc. This provides the superfield representation of the superalgebra (2.2).

The covariant derivatives D_A are superspace derivatives defined such that $D_A\phi$ transforms in the same way as the superfield ϕ itself. They are given by

$$
\begin{aligned}
D_a\phi &= \partial_a\phi\,, \\
D^i_\alpha\phi &= \left(\partial^i_\alpha + i\sigma^a_{\alpha\dot\beta}\bar\theta^{i\dot\beta}\partial_a - \theta^i_\alpha\partial_z\right)\phi\,, \qquad & \bar D_{i\dot\alpha}\phi &= \left(-\bar\partial_{i\dot\alpha} - i\theta^\alpha_i\sigma^a_{\alpha\dot\alpha}\partial_a + \bar\theta_{i\dot\alpha}\partial_{\bar z}\right)\phi\,, \\
D_z\phi &= \partial_z\phi\,, & D_{\bar z}\phi &= \partial_{\bar z}\phi\,.
\end{aligned}
\tag{2.5}
$$

and obey the same (anti)commutation rules as the generators, up to the sign of the right-hand sides:

$$
(D_A, D_B\} = -T^C_{AB}D_C\,, \tag{2.6}
$$

the torsion coefficients T^C_{AB} being given in (2.3).

The components of the supermultiplet corresponding to the superfield ϕ are the coefficients of its expansion in powers of θ and $\bar\theta$. A generic component can be written as

$$
C_n = (D)^n\phi|\,, \tag{2.7}
$$

where $(D)^n$ is some product of D^i_α and $\bar D_{i\dot\alpha}$ derivatives, and where the symbol $|$ means that the expression is evaluated at $\theta = \bar\theta = 0$. It follows from the explicit transformation rules (2.4), that the action of the supersymmetry generators on the components can be written as [9]

$$
Q^i_a C_n = D^i_\alpha(D)^n\phi\big|\,, \qquad \bar Q_{i\dot\alpha}C_n = D_{i\dot\alpha}(D)^n\phi|\,, \tag{2.8}
$$

3. The generalized central charge constraint for the free $N = 2$ Fayet hypermultiplet

The Fayet hypermultiplet

$$
\phi_i \equiv (\phi_i, \chi_\alpha, \bar\psi_{\dot\alpha}, F_i) \tag{3.1}
$$

is formed by two SU(2) doublets of Lorentz scalars $(\phi_i, F_{=}i)$ and two SU(2) singlets of Weyl spinors $\bar\psi_{\dot\alpha}$, χ_a. It will represent the matter sector of an N=2 supersymmetric Yang-Mills theory but, in the present section, we shall only consider a brief review of our previous work [11] on the free Fayet hypermultiplet. The latter is defined by an SU(2) doublet superfielkld[2] $\phi_i(X)$ subjected to the supersymmetric constraints [7, 8]

$$
D^i_\alpha\phi^j + D^j_\alpha\phi^i = 0\,, \qquad \bar D_{i\dot\alpha}\phi^j + \bar D^j_{\dot\alpha}\phi_i = 0\,. \tag{3.2}
$$

[2]The same symbol ϕ represents the multiplet (3.1), the corresponding superfield, as well as the first component of the latter, i.e. its value at $\theta = \bar\theta = 0$.

3.1 Central charge constraints and supersymmetry transformations

However, in order to define a finite supersymmetry representation, one has to impose a constraint which restricts the dependence of the superfield $\phi_i(\bar\phi^i)$ on the central charge coordinates z and $\bar z$. The constraint introduced in [11] reads

$$(\partial_z - \partial_{\bar z})\phi_i = i\Lambda\phi_i\,, \qquad (\partial_{\bar z} - \partial_z)\bar\phi^i = i\Lambda\bar\phi^i\,, \tag{3.3}$$

It is a generalization of Sohnius'one [9], introducing a new real parameter Λ of dimension of a mass.

Remark. The constraint actually considered in [11] had Λ complex[3] and involved a factor $\exp(-v)$, with v a real parameter, in front of the $\bar z$ derivative in the first equation (and of the z derivative in the second one). However it turned up that the existence of an invariant action implied the reality of Λ and the vanishing of v. This means that we would not be able to get a dynamics if Λ were complex or v nonvanishing, although we still would have a representation of the superalgebra (2.2). The reader may see [11] for more details.

Since the covariant derivatives D and $\bar D$ (2.5) commute with ∂_z and $\partial_{\bar z}$, the constraints above hold for the superfield $\phi_i(\bar\phi^i)$ and all its derivatives, in particular the derivatives which define the component fields (2.7).

Having defined the components of the hypermultiplet by

$$\phi_i \equiv \phi_i|\,, \quad \chi_a \equiv \tfrac{1}{2\sqrt{2}}D^i_\alpha\phi_i\Big|\,, \quad \bar\psi_{\dot\alpha} \equiv \tfrac{1}{2\sqrt{2}}\bar D^i_{\dot\alpha}\phi_i\Big|\,, \quad F^j \equiv \tfrac{i}{8}D^{i\alpha}D_{i\alpha}\phi^j\Big| = \partial_z\phi^j\ \Big|\,. \tag{3.4}$$

we found the following Λ-dependent supersymmetry and central charge transformation laws:

$$\begin{aligned}
Q^i_\alpha\phi_j &= \sqrt{2}\delta^i_j\chi_\alpha\,, & \bar Q_{i\dot\alpha}\phi^j &= -\sqrt{2}\delta^j_i\bar\psi_{\dot\alpha}\,,\\
Q^i_\alpha\chi_\beta &= -\sqrt{2}i\varepsilon_{\alpha\beta}F^i\,, & \bar Q_{i\dot\alpha}\chi_\alpha &= -\sqrt{2}i\partial_{\alpha\dot\alpha}\phi_i\,,\\
Q^i_\alpha\bar\psi_{\dot\beta} &= \sqrt{2}i\partial_{\alpha\dot\beta}\phi^i\,, & \bar Q_{i\dot\alpha}\bar\psi_{\dot\beta} &= \sqrt{2}i\varepsilon_{\dot\alpha\dot\beta}(F_i - i\Lambda\phi_i)\,,\\
Q^i_\alpha F_j &= \sqrt{2}\delta^i_j\left(\partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha} + i\Lambda\chi_\alpha\right), & \bar Q_{i\dot\alpha}F^j &= -\sqrt{2}\delta^j_i\partial_{\alpha\dot\alpha}\chi^\alpha\,,\\
Z\phi^i &= F^i\,, & \bar Z\phi^i &= (F^i - i\Lambda\phi^i)\,,\\
Z\chi_\alpha &= \partial_{\alpha\dot\beta}\bar\psi^{\dot\beta} + i\Lambda\chi_\alpha\,, & \bar Z\chi_\alpha &= \partial_{\alpha\dot\beta}\bar\psi^{\dot\beta}\,,\\
Z\bar\psi_{\dot\beta} &= \partial_{\alpha\dot\beta}\chi^\alpha\,, & \bar Z\bar\psi_{\dot\beta} &= (\partial_{\alpha\dot\beta}\chi^\alpha - i\Lambda\bar\psi_{\dot\beta})\,,\\
ZF^i &= \Box\phi^i + i\Lambda F^i\,, & \bar ZF^i &= \Box\phi^i\,,
\end{aligned} \tag{3.5}$$

and similarly for the conjugate components. The algebra of these transformations closes as a representation of the superalgebra (2.2).

3.2 The free Fayet lagrangian

The construction of an invariant lagrangian in [11] is based on a proposition due to Hasler [12]:

[3]Note that the present parameter Λ corresponds to $-i$ times the parameter λ of [11].

Proposition. Let be a superfield polynomial L^{ij} – called the "kernel" — satisfying the conditions of zero symmetric derivatives

$$D_\alpha^{(i}L^{jk)} = 0\,, \qquad \bar{D}_{\dot\alpha}^{(i}L^{jk)} = 0\,. \tag{3.6}$$

Then the superfields

$$L \equiv -D_k^\alpha \Lambda_\alpha^k\,, \qquad \bar{L} \equiv -\bar{D}_{k\dot\alpha}\bar{\Lambda}^{k\dot\alpha}\,, \tag{3.7}$$

where

$$\Lambda_\alpha^k \equiv D_{i\alpha}L^{ik}\,, \qquad \bar{\Lambda}^{k\dot\alpha} \equiv \bar{D}_i^{\dot\alpha}L^{ik}\,,$$

transform under supersymmetry — with infinitesimal parameters ξ, $\bar\xi$ — as

$$\begin{aligned}\delta L &= i\partial_z\left(\xi_i^\alpha\Lambda_\alpha^i + \bar\xi_{i\dot\alpha}\bar\Lambda^{i\dot\alpha}\right) - 2i\partial_a\left(\bar\xi_{i\dot\alpha}\bar\sigma^{a\,\dot\alpha\beta}\Lambda_\beta^i\right),\\ \delta\bar{L} &= -i\partial_{\bar z}\left(\xi_i^\alpha\Lambda_\alpha^i + \bar\xi_{i\dot\alpha}\bar\Lambda^{i\dot\alpha}\right) - 2i\partial_a\left(\xi_i^\alpha\sigma^a_{\alpha\dot\beta}\bar\Lambda^{idb}\right).\end{aligned} \tag{3.8}$$

Let us apply this proposition to the kernel used in [11],

$$L^{ij} = \partial_{\bar z}\bar\phi^j\phi^i + \bar\phi^i\partial_z\phi^j\,. \tag{3.9}$$

The central charge constraint (3.3) implies that $(\partial_z - \partial_{\bar z})L^{ij}$ is a total space-time derivative. Then the free lagrangian defined by

$$\mathcal{L}_{\text{free}} = \frac{-1}{24}\left(L + \bar{L}\right), \tag{3.10}$$

L being defined by (3.7), is indeed supersymmetry invariant up to a total space-time derivative. Explicitly[4]:

$$\mathcal{L}_{\text{free}} = \bar{F}F - \partial_a\bar\phi\partial^a\phi - i\chi\partial\!\!\!/\bar\chi - i\psi\partial\!\!\!/\bar\psi - i\frac{\Lambda}{2}\left(\bar{F}\phi + i\bar\chi\bar\psi + i\psi\chi - \bar\phi F\right), \tag{3.11}$$

with the notation $(\partial\!\!\!/\bar\psi)_\alpha = \partial_{\alpha\dot\beta}\bar\psi^{\dot\beta} = \sigma^a_{\alpha\dot\beta}\partial_a\bar\psi^{\dot\beta}$.

The terms in Λ are mass terms, which have been induced from the supersymmetry transformation rules written above. Writing down the equations of motion [11] indeed shoes that $\Lambda/2$ represents the value of the mass of the propagating fields ϕ^i, χ and ψ, the field F^i being auxiliary.

Of course, it is still possible to add a mass term "by hand". This can be done with the help of Hasler's proposition, too, and leads to the independent invariant mass lagrangian

$$\mathcal{L}_\mu = \mu\left(\bar{F}\phi + \bar\phi F + i\bar\chi\bar\psi - i\psi\chi\right), \tag{3.12}$$

where μ is a real mass parameter.

However, imposing invariance under the parity transformations[5]

$$\begin{aligned}(x^0, \mathbf{x}) &\to (x^0, -\mathbf{x})\,,\\ (\phi^i, \chi^\alpha, \bar\psi_{\dot\beta}, F^i) &\leftrightarrow \left(\bar\phi_i, \bar\chi_{\dot\alpha}, \psi^\beta, -\bar{F}_i\right),\end{aligned} \tag{3.13}$$

rules out the mass lagrangian (3.12). Thus parity invariance insures that the mass is completely determined by the parameter Λ.

[4] See (A.3) for the summation conventions.

[5] The 4-component Dirac spinor $\Psi = (\chi_\alpha,\, \bar\psi^{\dot\alpha})$ then transforms as $\Psi \to \gamma^0\Psi$.

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4. Coupling with an $N = 2$ gauge supermultiplet

4.1 Gauge transformations and covariant derivatives in superspace

The construction of N=2 supersymmetric Yang-Mills theory is based on a SU(2) doublet of Fayet hypermultiplets of matter fields, described by the superfields ϕ^i, $i = 1, 2$ and now belonging to some representation R of a compact Lie group G, the gauge group. The conjugate superfields ϕ_i belong to the conjugate representation $\bar{R}$. These superfields are subjected to a generalization of the constraints shown in the previous section, and which will be introduced in the next subsection. The generators of G in the representation R are antihermitean matrices T_r obeying the Lie algebra commutation rules

$$[T_r, T_s] = f_{rs}{}^t T_t \,. \tag{4.1}$$

Local gauge invariance requires the introduction of a gauge connection superfield

$$\Phi_A = \Phi^r_A T_r \,, \qquad A = a, i\alpha, i\dot{\alpha}, z, \bar{z} \,. \tag{4.2}$$

with the antihermicity conditions

$$(\Phi^a)^+ = -\Phi^a \,, \qquad (\Phi^i_\alpha)^+ = -\Phi_{i\dot{\alpha}} \,, \qquad (\Phi_z)^+ = +\Phi_{\bar{z}} \,. \tag{4.3}$$

The infinitesimal gauge transformations read

$$\delta\phi^i = \Omega\phi^i \,, \qquad \delta\bar{\phi}_i = \bar{\phi}_i\Omega \,, \qquad \delta\Phi_A = D_A\Omega + [\Omega, \Phi_A] \,, \tag{4.4}$$

where the infinitesimal parameter $\Omega = \Omega^r T_r$ is an antihermitean superfield, subjected to some restrictions because of the constraints on the Fayet superfield ϕ^i, as it will be shown in the next subsection. The covariant derivatives

$$\mathcal{D}_A\phi^i = D_A\phi^i - \Phi_A\phi^i \,, \qquad \mathcal{D}_A\bar{\phi}_i = D_A\bar{\phi}_i + \bar{\phi}_i\Phi_A \,, \tag{4.5}$$

with D_A the ordinary superspace covariante derivative (2.5), transform in the same way as ϕ and $\bar{\phi}$ in (4.4). Note that $\bar{\phi}^i\phi_i$ being gauge invariant, we have

$$\mathcal{D}_A\left(\bar{\phi}^i\phi_i\right) = D_A\left(\bar{\phi}^i\phi_i\right) \,. \tag{4.6}$$

The super Yang-Mills curvature [1]

$$\mathcal{F}_{AB} = T^C_{AB}\Phi_C + D_A\Phi_B - (-)^{bc}D_B\Phi_A - (\Phi_A, \Phi_B\} \,, \tag{4.7}$$

where the torsion coefficients T^C_{AB} are the same as in the free case (2.3), transforms covariantly, in the adjoint representation:

$$\delta\mathcal{F}_{AB} = [\Omega, \mathcal{F}_{AB}] \,. \tag{4.8}$$

Its covariant derivative is

$$\mathcal{D}_A\mathcal{F}_{BC} = D_A\mathcal{F}_{BC} - (\Phi_A, \mathcal{F}_{BC}\} \,. \tag{4.9}$$

where we recall that the symbol $(\cdot, \cdot\}$ is an anticommutator if both entries are fermionic, and a commutator otherwise. The commutation rules of the gauge covariant superspace derivatives (4.5) or (4.9) yields the supercurvature, e.g.:

$$(\mathcal{D}_A, \mathcal{D}_B\}\,\Phi_D = -T^C_{AB}\mathcal{D}_C\Phi_D - (\mathcal{F}_{AB}, \Phi_D\} \,. \tag{4.10}$$

4.2 The gauge supermultiplet

As usual [1, 8, 3], the gauge superfields Φ_A must be constrained in order to get a sensible gauge theory. In the present case of central charge superspace, the natural constraints consist in the vanishing of all the supercurvature components with spinor indices [10, 4, 5]:

$$\mathcal{F}^{i\,j}_{\alpha\beta} = \mathcal{F}^{i\,j}_{\dot\alpha\dot\beta} = \mathcal{F}^{i\,j}_{\alpha\dot\beta} = 0\,. \tag{4.11}$$

Adding the constraint of central charge independence for the gauge superfields:

$$\partial_z \Phi_A = \partial_{\bar z}\Phi_A = 0\,, \tag{4.12}$$

one can use the Bianchi identities [8]

$$\sum_{\text{cyclic}\,(ABC)} \left(\mathcal{D}_A \mathcal{F}_{BC} - T^E_{AB}\mathcal{F}_{CE}\right) = 0\ , \tag{4.13}$$

in order to show [4] that all the nonvanishing curvature components may be expressed in terms of the gaugino superfield Φ^i_α and its conjugate:

$$\begin{aligned}
\mathcal{F}^{i\ a}_{\alpha} &= i\sigma^a_{\alpha\dot\alpha}\Phi^{i\dot\alpha}\,, \qquad \mathcal{F}^{\dot\alpha\ a}_{i} = -i\bar\sigma^{a\dot\alpha\alpha}\Phi^i_\alpha\ ,\\
\mathcal{F}_{z\ i}^{\ \dot\alpha} &= -4\Phi^{\dot\alpha}_i\ , \qquad \mathcal{F}_{\bar z\ \alpha}^{\ i} = 4\Phi^i_\alpha\ ,\\
\mathcal{F}_{a\,b} &= \frac{i}{16}\left(\sigma^{\alpha\beta}_{ab}\mathcal{D}_{\alpha\beta} + \bar\sigma^{\dot\alpha\dot\beta}_{ab}\mathcal{D}_{\dot\alpha\dot\beta}\right)\ , \quad \text{with} \quad \mathcal{D}_{\alpha\beta} \equiv \left(\mathcal{D}^k_\alpha\mathcal{D}_{k\beta} + \mathcal{D}^k_\beta\mathcal{D}_{k\alpha}\right),\\
\mathcal{F}_{z\,\bar z} &= -\left[\phi_z, \phi_{\bar z}\right) \qquad .
\end{aligned} \tag{4.14}$$

Gaida [10] has shown that all curvature components only depend on the components Φ_z and $\Phi_{\bar z}$ of the gauge connection, through the identities

$$\Phi^i_\alpha = -\mathcal{D}^i_\alpha\Phi_{\bar z}\,, \qquad \Phi^{\dot\alpha}_i = -\mathcal{D}^{\dot\alpha}_i\Phi_z \tag{4.15}$$

In order to keep consistence with the condition (4.12), we must take the infinitesimal parameter Ω of the gauge transformations (4.4) independent of z and $\bar z$. We then note that Φ_z and $\Phi_{\bar z}$ transform covariantly under the gauge transformations. Moreover, due to (4.13) and (4.14), they obey to the covariantized chirality conditions

$$\mathcal{D}^i_{\dot\alpha}\Phi_{\bar z} = 0\,, \qquad \mathcal{D}^i_\alpha\Phi_z = 0\ . \tag{4.16}$$

Supersymmetry transformations of the chiral superfield $\Phi_{\bar z}$ generate the components of the gauge supermultiplet, which consists of a scalar X, an SU(2) doublet, spinor X^i_α (the gaugino), an SU(2) triplet $X^{(ij)}$ and a Lorentz triplet $X_{\alpha\beta}$, defined by

$$\begin{aligned}
X &= -\ \Phi_{\bar z}|\,,\\
X^i_\alpha &= -\ \mathcal{D}^i_\alpha\Phi_{\bar z}\big|\,,\\
X^{(ij)} &= -\ \mathcal{D}^{ij}\Phi_{\bar z}\big|\,, \qquad \text{with} \quad \mathcal{D}^{ij} \equiv \mathcal{D}^{i\alpha}\mathcal{D}^j_\alpha + \mathcal{D}^{j\alpha}\mathcal{D}^i_\alpha\,,\\
X_{(\alpha\beta)} &= -\ \mathcal{D}_{\alpha\beta}\Phi_{\bar z}|\,, \qquad \text{with} \quad \mathcal{D}_{\alpha\beta} \equiv \left(\mathcal{D}^k_\alpha\mathcal{D}_{k\beta} + \mathcal{D}^k_\beta\mathcal{D}_{k\alpha}\right).
\end{aligned} \tag{4.17}$$

The SU(2) triplet obeys the condition of reality:

$$\left(X^{ij}\right)^{+} = -(X_{ij})^{+} .$$

The Lorentz triplet and its conjugate are linked to the Yang-Mills curvature

$$\mathcal{F}_{ab} = \partial_a A_b - \partial_b A_a - [A_a, A_b] , \qquad \text{where} \;\; A_a = \Phi_a|$$

by:

$$X_{\alpha\beta} = 8i\sigma^{ab}_{\alpha\beta}\mathcal{F}_{ab} , \qquad \bar{X}_{\dot\alpha\dot\beta} = 8i\bar\sigma^{ab}_{\dot\alpha\dot\beta}\mathcal{F}_{ab} ,$$

or, conversely:

$$\mathcal{F}^{ab} = -\frac{i}{16}\left(\sigma^{ab}_{\alpha\beta}X^{\alpha\beta} + \sigma^{ab}_{\dot\alpha\dot\beta}\bar{X}^{\dot\alpha\dot\beta}\right) , \tag{4.18}$$

The transformation laws of the gauge supermultiplet are obtained using the definition (4.17) of the components and Equation (2.8) with the ordinary superspace spinor derivatives D^i_α, $\bar{D}^i_{\dot\alpha}$ replaced by the covariant derivatives $\mathcal{D}^i_\alpha$, $\mathcal{D}^i_{\dot\alpha}$. The result is:

$$\begin{aligned}
Q^i_\alpha X &= X^i_\alpha \\
Q^i_\alpha X^j_\beta &= -i\varepsilon^{ij}\varepsilon_{\alpha\beta}\left[\bar{X}, X\right] + \frac{1}{4}\varepsilon_{\alpha\beta}X^{ij} - \frac{1}{4}\varepsilon^{ij}X_{\alpha\beta} \\
Q^i_\alpha X^{jk} &= 4i\mathcal{D}_{\alpha\dot\beta}(\varepsilon^{ij}\bar{X}^{\dot\beta k} + \varepsilon^{ik}\bar{X}^{\dot\beta j}) - 4i\left[\varepsilon^{ij}X^k_\alpha + \varepsilon^{ik}X^j_\alpha, \bar{X}\right] \\
Q^i_\alpha X_{\beta\gamma} &= 4i(\varepsilon_{\alpha\beta}\mathcal{D}_{\gamma\dot\alpha} + \varepsilon_{\alpha\gamma}\mathcal{D}_{\beta\dot\alpha})\bar{X}^{i\dot\alpha} \\
\bar{Q}_{k\dot\alpha}X &= 0 \\
\bar{Q}_{k\dot\alpha}X^j_\beta &= -2i\delta^j_k\mathcal{D}_{\beta\dot\alpha}X \\
\bar{Q}_{k\dot\alpha}X^{ij} &= 4i\mathcal{D}_{\beta\dot\alpha}(\delta^i_k X^{j\beta} + \delta^j_k X^{i\beta}) - 4i\left[\delta^i_k\bar{X}^j_{\dot\alpha} + \delta^j_k\bar{X}^i_{\dot\alpha}, X\right] \\
\bar{Q}_{k\dot\alpha}X_{\alpha\beta} &= -4i(\mathcal{D}_{\alpha\dot\alpha}X_{k\beta} + \mathcal{D}_{\beta\dot\alpha}X_{k\alpha})
\end{aligned} \tag{4.19}$$

and similar transformations for the conjugated multiplet $\bar{X} = (\bar{X}, \bar{X}_{i\dot\alpha}, X_{ij}\bar{X}_{\dot\alpha\dot\beta})$. In the equations above, $\mathcal{D}_{\beta\dot\alpha} \equiv \sigma^a_{\beta\dot\alpha}\mathcal{D}_a = \sigma^a_{\beta\dot\alpha}(\partial_a - [A_a,\ ])$.

4.3 Gauge lagrangian

The gauge field supermultiplet being chiral, the corresponding gauge invariant lagrangian, supersymmetric up to a total derivative, may be defined as

$$\mathcal{L}_{\text{gauge}} = \frac{1}{3\cdot 2^9}\mathrm{Tr}\,\left(\mathcal{D}^{ij}\mathcal{D}_{ij}(\Phi_{\bar z})^2 + \text{c.c.}\right) , \tag{4.20}$$

with $\mathcal{D}^{ij}$ defined in (4.17). Using the transformations laws (4.19) and equation (4.18) we explicitly get

$$\begin{aligned}
\mathcal{L}_{\text{gauge}} = \frac{1}{4}\mathrm{Tr}\,\mathcal{D}^a\mathcal{D}_a\bar{X}X + \frac{1}{4}\bar{X}\mathcal{D}_a\mathcal{D}^aX - \frac{i}{8}(\mathcal{D}_{\alpha\dot\alpha}\bar{X}^{i\dot\alpha})X^\alpha_i + \frac{i}{8}\bar{X}^{\dot\alpha i}(\mathcal{D}_{\alpha\dot\alpha}X^{i\alpha}) - \\
-\frac{1}{4}\mathcal{F}_{ab}\mathcal{F}^{ab} + \frac{1}{256}X^{ij}X_{ij} + \frac{i}{8}\bar{X}\{X^{\alpha i}, X_{\alpha i}\} + \frac{i}{8}\{\bar{X}^i_{\dot\alpha}, \bar{X}^{\dot\alpha}_i\}X + \frac{1}{8}[X,\bar{X}]\bar{X}X .
\end{aligned} \tag{4.21}$$

4.4 Generalized Fayet-Sohnius constraints. Supersymmetry transformations of the hypermultiplet

The natural extension of the condition (3.2) defining ϕ^i as a Fayet hypermultiplet, in the presence of a gauge connection, is given by:

$$\mathcal{D}^i_\alpha\phi^j + \mathcal{D}^j_\alpha\phi^i = 0\,, \qquad \mathcal{D}^i_{\dot\alpha}\phi^j + \mathcal{D}^j_{\dot\alpha}\phi^i = 0\,, \tag{4.22}$$

and the conjugate equations. The use of the covariant derivative guarantees the compatibility of this condition with gauge covariance.

For what concerns the generalized central charge constraint on the Fayet superfield, necessary in order to get a finite supermultiplet, we shall use the same condition (3.3) as in the free case:

$$(\partial_z - \partial_{\bar z})\phi_i = i\Lambda\phi_i\,, \qquad (\partial_{\bar z} - \partial_z)\bar\phi^i = i\Lambda\bar\phi^i\,. \tag{4.23}$$

The compatibility of this constraint with gauge covariance relies on the fact that the z and $\bar z$ components of the connection, namely $\bar X$ and x (see (4.17)), are covariant, which implies the covariance of the partial derivatives $\partial/\partial z$ and $\partial/\partial\bar z$. Let us mention that a slightly more general condition is possible here, too, as we have noted in section 3 for the free case, but which as well in the present interactive case would prejudicate the existence of an invariant action.

The definition of the hypermultiplet components in the presence of the gauge connection is a covariant extension of the definition (3.4) proposed in the free case:

$$\begin{aligned} \phi_i &= \phi_i|\,, \qquad \chi_a = \frac{1}{2\sqrt2}\mathcal{D}^i_\alpha\phi_i\Big|\,, \qquad \bar\psi_{\dot\alpha} = \frac{1}{2\sqrt2}\bar{\mathcal{D}}^i_{\dot\alpha}\phi_i\Big|\,,\\ F^j &= \partial_z\phi^j\;\Big| = \left(\frac{i}{8}\mathcal{D}^{i\alpha}\mathcal{D}_{i\alpha}\phi^j + \Phi_z\phi^j\right)\Big| \end{aligned} \tag{4.24}$$

and similarly for the conjugated hypermultiplet $\bar\phi^i$. Let us note that the component F^i is defined as the simple partial derivative $\partial/\partial z$ instead of the covariant one. The latter and its conjugate then read, at $\theta = 0$,

$$\begin{aligned} \mathcal{D}_z\phi^i\big| &= (\partial_z - \Phi_z)\phi^i\big| = F^i + \bar X\phi^i\\ \mathcal{D}_{\bar z}\phi^i\big| &= (\partial_{\bar z} - \Phi_{\bar z})\phi^i\big| = F^i + (X - i\Lambda)\phi^i\,. \end{aligned} \tag{4.25}$$

The supersymmetry transformations of the components are then defined by the covariantized expressions (2.8). Using the definitions above, the curvature conditions (4.11), the Fayet constraints (4.22) and the central charge constraints (4.23) we find, with the help of the Bianchi identities (4.13), the following supersymmetry transformation laws of the hypermultiplet:

$$\begin{aligned} Q^i_\alpha\phi_j &= \sqrt2\delta^i_j\chi_\alpha\,, & \bar Q_{i\dot\alpha}\phi^j &= -\sqrt2\delta^j_i\bar\psi_{\dot\alpha}\,,\\ Q^i_\beta\chi_\alpha &= -\sqrt2 i\varepsilon_{\beta\alpha}(F^i + \bar X\phi^i)\,, & \bar Q_{i\dot\alpha}\chi_\beta &= -\sqrt2 i\mathcal{D}_{\beta\dot\alpha}\phi_i\,,\\ Q^i_\beta\bar\psi_{\dot\beta} &= \sqrt2 i\mathcal{D}_{\beta\dot\beta}\phi^i\,, & \bar Q_{i\dot\alpha}\bar\psi_{\dot\beta} &= \sqrt2 i\varepsilon_{\dot\alpha\dot\beta}\left(F_i - (i\Lambda - X)\phi_i\right), \end{aligned}$$

$$\begin{aligned} Q^i_\alpha F_j &= \sqrt2\delta^i_j\left(\mathcal{D}_{\alpha\dot\alpha}\bar\psi^{\dot\alpha} + (i\Lambda - X)\chi_\alpha - X^k_\alpha\phi_k\right),\\ \bar Q_{i\dot\alpha}F^j &= -\sqrt2\delta^j_i\left(\mathcal{D}_{\alpha\dot\alpha}\chi^\alpha - \bar X\bar\psi_{\dot\alpha} + \bar X^k_{\dot\alpha}\phi_k\right). \end{aligned} \tag{4.26}$$

In the same way one finds the central charge transformations

$$
\begin{aligned}
&Z\phi^i = F^i + \bar{X}\phi^i\,, \qquad \bar{Z}\phi^i = F^i - (i\Lambda - X)\phi^i \\
&Z\chi_\alpha = \mathcal{D}_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} + (i\Lambda - X + \bar{X})\chi_\alpha - X^k_\alpha\phi_k\,, \qquad \bar{Z}\chi_\alpha = \mathcal{D}_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} - \tfrac{1}{\sqrt{2}}X^k_\alpha\phi_k\,, \\
&Z\bar{\psi}_{\dot{\alpha}} = \mathcal{D}_{\alpha\dot{\alpha}}\chi^\alpha + \bar{X}^k_{\dot{\alpha}}\phi_k\,, \qquad \bar{Z}\bar{\psi}_{\dot{\alpha}} = \mathcal{D}_{\alpha\dot{\alpha}}\chi^\alpha + \bar{X}^k_{\dot{\alpha}}\phi_k - (i\Lambda - X + \bar{X})\bar{\psi}_{\dot{\alpha}}\,, \\
&ZF^i = \Box\phi^i + (i\Lambda + \bar{X})F^i\,, \qquad \bar{Z}F^i = \Box\phi^i + XF^i
\end{aligned}
\tag{4.27}
$$

and similar transformations for the conjugated hypermultiplet $\bar{\phi}^i$. In these equations, $\mathcal{D}_{\alpha\dot{\alpha}} = \sigma^a_{\alpha\dot{\alpha}}\mathcal{D}_a = \sigma^a_{\alpha\dot{\alpha}}(\partial_a - A_a)$.

4.5 The hypermultiplet lagrangian minimally coupled to the gauge connection

The construction of the hypermultiplet lagrangian minimally coupled to the gauge connection follows the same lines as in the free case, being based on Hasler's proposition stated in section 3.2. We first observe that the kernel L^{ij} (3.9) is gauge invariant as it reads. This way, we have

$$
D_A L^{ij} = \mathcal{D}_A L^{ij}\,, \quad \text{for all A}\,,
\tag{4.28}
$$

and so, the natural extension of Hasler's procedure[6] in order to get a gauge invariant lagrangian, supersymmetry invariant up to a total derivative, is to substitute in every step the ordinary superspace derivative D_A used in the free case, by the gauge covariant derivatives $\mathcal{D}_A$, using the property

$$
D_A \mathrm{Tr}\,(\bar{\varphi}\varphi') = \mathrm{Tr}\,\left(\mathcal{D}_A\bar{\varphi}\varphi' + \bar{\varphi}\mathcal{D}_A\varphi'\right)\,,
\tag{4.29}
$$

φ and φ' being the Fayet superfield or some of its covariant derivatives. Since the latter define the components of the hypermultiplet and their transformation rules through (4.24) and the covariantized form of (2.8), it is easy to compute the lagrangian using the supersymmetry transformation laws (4.26). Thus, starting with the kernel (3.9), we get

$$
\begin{aligned}
\mathcal{L}_{\text{hypermultiplet}} = {}& \bar{F}F - i\psi^\alpha\mathcal{D}_{\alpha\dot{\beta}}\bar{\psi}^{\dot{\beta}} + i\bar{\chi}^{\dot{\beta}}\mathcal{D}_{\alpha\dot{\beta}}\chi^\alpha - \mathcal{D}^a\bar{\phi}\mathcal{D}_a\phi + \\
&+ i\psi X\chi - i\bar{\chi}\bar{X}\bar{\psi} + \frac{i}{\sqrt{2}}\psi^\alpha X^k_\alpha\phi_k - \frac{i}{\sqrt{2}}\bar{\phi}^k\bar{X}_{\dot{\alpha}k}\bar{\psi}^{\dot{\alpha}} - \frac{i}{\sqrt{2}}\bar{\phi}^k X_{k\alpha}\chi^\alpha - \\
&- \frac{i}{\sqrt{2}}\bar{\chi}_{\dot{\alpha}}\bar{X}^{\dot{\alpha}k}\phi_k - \frac{1}{2}\bar{\phi}(\bar{X}X + X\bar{X})\phi + \frac{i}{8}\bar{\phi}^i X_{ij}\phi^j - \\
&- \frac{i}{2}\Lambda\left(\bar{F}\phi - \bar{\phi}F + i\psi\chi + i\bar{\chi}\bar{\psi} - \bar{\phi}\bar{X}\phi + \bar{\phi}X\phi\right)\,,
\end{aligned}
\tag{4.30}
$$

As in the free case, an independent invariant mass lagrangian may be added, in the form of the gauge invariant extension of the supersymmetric mass lagrangian (3.12):

$$
\mathcal{L}_\mu = \mu\left(\bar{F}\phi + \bar{\phi}F + i\bar{\chi}\bar{\psi} - i\psi\chi + \bar{\phi}X\phi + \bar{\phi}\bar{X}\phi\right)\,,
\tag{4.31}
$$

with μ a real mass parameter. However this mass lagrangian again is ruled out by the

[6] This procedure is detailed in [11].

requirement of invariance under the parity transformations

$$
\begin{aligned}
\left(x^0,\mathbf{x}\right) &\rightarrow \left(x^0,-\mathbf{x}\right), \\
\left(\phi^i,\chi^\alpha,\bar\psi_{\dot\beta},F^i\right) &\leftrightarrow \left(\bar\phi_i,\bar\chi_{\dot\alpha},\psi^\beta,-\bar F_i\right), \\
\left(X,\bar X,X^k_\alpha,\bar X^{\dot\alpha}_k,X^{ij}\right) &\rightarrow \left(-\bar X^{\mathrm T},-X^{\mathrm T},-\left(\bar X^{\mathrm T}\right)^{\dot\alpha}_k,-\left(X^{\mathrm T}\right)^k_\alpha,\left(X^{\mathrm T}\right)_{ij}\right)
\end{aligned}
\tag{4.32}
$$

where the superscript T means transposição.

In order to complete the description of the model, we write down the field equations for the matter fields:

$$
\begin{aligned}
F_i - iM\phi_i &= 0\,, \\
\mathcal{D}_a\mathcal{D}^a\phi_i + iMF_i + \left(iM(\bar X - X) - \frac{1}{2}(\bar X X + X\bar X)\right)\phi_i - & \\
-\frac{i}{\sqrt2}(\bar X_{i\dot\alpha}\bar\psi^{\dot\alpha} + X_{i\alpha}\chi^\alpha)\phi_i + \frac{i}{8}X_{ik}\phi^k &= 0\,, \\
-i\mathcal{D}_{\alpha\dot\beta}\bar\psi^{\dot\beta} + (M+iX)\chi_\alpha + \frac{i}{\sqrt2}X^k_\alpha\phi_k &= 0\,, \\
i\mathcal{D}^{\alpha\dot\beta}\chi_\alpha - (M+i\bar X)\bar\psi^{\dot\beta} - \frac{i}{\sqrt2}\bar X^{\dot\beta k}\phi_k &= 0
\end{aligned}
\tag{4.33}
$$

where the mass M is given by

$$
M = \frac{\Lambda}{2}\,.
$$

5. Conclusions

We have shown that the generalized central charge constraint proposed in [11] for the free Fayet hipermultiplet is working in the case of minimal coupling with a super-Yang-Mills connection. In this case, too, the constraint modifies the supersymmetry transformation rules with terms depending on a parameter having the dimension of a mass. This parameter indeed shows up in the resulting action as the mass of the hypermultiplet. Moreover the mass is totally induced by this mechanism if parity invariance is imposed, which lets us conclude to the possibility of a nonrenormalization theorem for the mass.

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A. Notations and conventions

Space-time is Minkovskian, 4-vector components are labelled by latin letters $a, b, \cdots = 0, 1, 2, 3$, the metric is choosen as

$$\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)\,. \tag{A.1}$$

Weyl spinors are complex 2-component spinors ψ_α, $\alpha = 1, 2$, in the $(\frac{1}{2}, 0)$ representation of the Lorentz group, or $\psi_{\dot\alpha}$, $\dot\alpha = 1, 2$, in the $(0, \frac{1}{2})$ representation. The $N = 2$ internal symmetry group is "isospin" SU(2), isospinors being denoted by X^i, $i = 1, 2$.

Isospin indices i are raised and lowered by the antisymmetric tensors ε^{ij} and ε_{ij}:

$$\begin{aligned} X^i &= \varepsilon^{ij} X_j\,, \qquad X_i = \varepsilon_{ij} X^j\,, \\ \text{with:}\quad \varepsilon^{ij} &= -\varepsilon^{ji}\,, \quad \varepsilon^{12} = 1\,, \quad \varepsilon_{ij}\varepsilon^{jk} = \delta_i^k\,, \quad \varepsilon^{ij}\varepsilon_{kl} = \delta_l^i\delta_k^j - \delta_k^i\delta_l^j\,. \end{aligned} \tag{A.2}$$

The same holds for the Lorentz spin indices, with the tensors $\varepsilon^{\alpha\beta}$ and $\varepsilon^{\dot\alpha\dot\beta}$ obeying to the same rules (A.2).

Multiplication of spinors and isospinors is done, if not otherwise stated, according to the convention

$$\psi\chi = \psi^\alpha\chi_\alpha\,, \qquad \bar\psi\bar\chi = \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha}\,, \qquad UV = U^i V_i\,. \tag{A.3}$$

Our conventions for the complex conjugation, denoted by *, are as follows:

$$(X_\alpha^i)^* = \bar X_{i\dot\alpha}\,, \qquad (\bar X_{i\dot\alpha})^* = X_\alpha^i\,. \tag{A.4}$$

The matrices σ^a and $\bar\sigma^a$ are defined by

$$\begin{aligned} \bar\sigma^{a\,\dot\alpha\alpha} &= \varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}\sigma^a_{\beta\dot\beta}\,, \\ \sigma^0 &= \mathbb{1}\,, \quad \sigma^i\,(i = 1, 2, 3) = \text{Pauli matrices}\,, \\ \bar\sigma^0 &= \mathbb{1}\,, \quad \bar\sigma^i\,(i = 1, 2, 3) = -\text{Pauli matrices}\,, \end{aligned} \tag{A.5}$$

and obey the properties

$$\sigma^a\bar\sigma^b + \sigma^b\bar\sigma^a = -2\eta^{ab}\,, \qquad \sigma^a_{\alpha\dot\alpha}\bar\sigma_a^{\dot\beta\beta} = -2\delta_\alpha^\beta\delta_{\dot\alpha}^{\dot\beta}\,. \tag{A.6}$$

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